DYNAMIC MODEL DESCRIBING RESPONSE OF GLASS-FIBER EXTRUSION PROCESS TO EXTERNAL PERTURBATIONS

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A model is proposed for describing the dynamics of glass-fiber extrusion, and on its basis are determined the amplitude—frequency characteristics of the produced fiber cross-section, depending on technological perturbations. The effect of viscous relaxation on the magnitude of residual stresses in a multilayer optical fiber is also evaluated on this basis.

1. Imposing stringent constraints on the nonuniformity of parameters of an optical glass fiber so as to minimize the power lost by light beams propagating through it requires a mathematical model of the fiber-forming process. Nonuniformity of the parameters of an extruded fiber is a consequence of the system response to various external perturbations influencing the extrusion process; hence mathematical models describing this response are a matter of utmost interest.

In this report will be presented results of a study concerning the effect of perturbations on the characteristics of an extruded fiber. Usually the extrusion of a glass fiber is described in terms of a problem in hydrodynamics for jet flow of an incompressible Newtonian fluid with a temperature-dependent viscosity, this dependence as well as the temperature distribution assumed to be known. Unlike in earlier studies (e.g., [1-3]), where the Navier—Stokes equation was used as the point of departure, here we will use methods of nonlinear thermomechanics [4] which have yielded the dimensionless equation of motion of a fiber during its deformation:

$$y_{zz}y_t - y_{zt}y_z = \eta\lambda(t)y_z, \tag{1}$$

with the function y(z, t) of one space coordinate and of time called "movement," η(z, t) denoting the fluidity, and the arbitrary function λ(t) of time only determined from the boundary conditions (subscripts denote differentiation with respect to the corresponding variable).

The observable dimensionless variables, namely velocity v and fiber cross section s, can be expressed through "movement" according to the relations

$$s = y_z;\ v = -\frac{y_t}{y_z}. \tag{2}$$

We will consider only weak irregularities in the process, those which do not drastically alter the conditions of extrusion in the steady state, and therefore will apply the methods of perturbation theory to small deviations from steady "movement"

$$y_0 = \int_0^z e^{-w\zeta(x)}\,dx - t;\ \left(w = \ln V_b,\ \zeta(x) = \int_0^x \eta(s)\,ds\right), \tag{3}$$

caused by small transient changes in the problem input parameters. We thus arrive at the equation for the correction P(z, t) to steady motion, in the first approximation,

$$P_{zz} + w\eta P_z + e^{-w\zeta(z)}P_{zt} + w\eta e^{-w\zeta(z)}P_t = -(wv + \mu(t))\,e^{-w\zeta(z)} \tag{4}$$

under boundary conditions

$$\text{a) } P(z,\ 0) = 0,$$
$$\text{b) } P_z(0,\ t) = \varphi(t),$$

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$$\text{c) } P_t(0,\ t) = -\psi(t) - \varphi(t),$$
$$\text{d) } P_t(1,\ t) = e^{w} P_z(1,\ t) - e^{-w}\chi(t). \quad (5)$$

Here $\nu(z,\ t)$ is the fluidity perturbation, $\varphi(t)$ is the perturbation of initial cross section, $\psi(t)$ is the perturbation of initial velocity, $\chi(t)$ is the perturbation of the extrusion rate, and $\mu(t)$ is an arbitrary function of time determined from boundary condition (5d).

Problem (4)-(5) can be solved analytically for the case of uniform fluidity along the deformation zone ($\eta = 1$ for $z \in [0,\ 1]$). For this case there have been considered four kinds of prolems:

I) perturbation of the steady state by small transient changes of initial velocity ($\nu=0,\ \varphi=0,\ \chi=0$);

II) perturbation of the steady state by small transient changes of the extrusion rate ($\nu=0,\ \varphi=0,\ \psi=0$);

III) perturbation of the steady state by small transient changes of initial cross section ($\nu=0,\ \psi=0,\ \chi=0$);

IV) perturbation of the steady state by small transient changes of viscosity ($\varphi=0,\ \psi=0,\ \chi=0$).

The general solution to Eq. (4) is

$$P = e^{-wz}\left\{\int_0^z\left[f\left(\frac{1}{w}e^{-wx}+t\right) + \int_t^{\frac{1}{w}e^{-wx}+t}\left(\mu(\xi) + w\nu\left(-\frac{1}{w}\ln\left(e^{-wx} - w(\xi - t)\right),\ \xi\right)\right)d\xi\right]e^{wx}dx + c(t)\right\}. \quad (6)$$

Solution of any of the problems I-IV reduces to determination of the arbitrary functions f, c, and μ in the general solution from the boundary conditions (5). Function $\mu(t)$ can, by virtue of condition (5d), be calculated from the integral equation

$$\int_0^t \mu(s)\,k(t-s)\,ds - \frac{1}{w}(e^{w}-1)\,\mu(t) = f(t), \quad (7)$$

where f(t) is an expression describing one of the perturbations and

$$k(s) = \begin{cases} \dfrac{1-e^{w}(1-ws)}{(1-ws)^2} & s \in \left[0,\ \dfrac{1}{w}(1-e^{w})\right], \\ 0 & s \in \left[\dfrac{1}{w}(1-e^{-w}),\ \infty\right]. \end{cases}$$

Most attention was paid to periodic perturbations of the $a\sin\omega t$ form. Of practical interest in this case are the amplitude—frequency characteristics of the system reaction to these perturbations, viz., the ratio of the amplitude of relative variations of the extruded fiber cross section to the relative amplitude of perturbations as a function of the perturbation frequency ω. Closed analytical expressions describing those amplitude—frequency characteristics have been obtained for all four kinds of problems.

As an example we will show here one of the simplest characteristics, describing the amplitude of variations of the fiber cross section at the exit from the deformation zone as a function of the frequency of extrusion rate fluctuations:

$$A(\omega) = a\frac{e^{-w}}{M^2+L^2}\sqrt{(MC+LS)^2+(MS+LC)^2}, \quad (8)$$

with the notation

$$L = b\left[S\cos(\Omega-\varphi) - (C\sin(\Omega-\varphi)\right] + e^{w}(\cos\Omega l - 1);$$
$$M = b\left[S\sin(\Omega-\varphi) + C\cos(\Omega-\varphi)\right] + e^{w}\sin\Omega l;$$
$$\Omega = \omega/w;\ l = 1 - e^{-w},\ S = \mathrm{si}\,\Omega e^{-w} - \mathrm{si}\,\Omega;\ C = \mathrm{ci}\,\Omega e^{-w} - \mathrm{ci}\,\Omega;$$
$$b = \sqrt{\Omega^2 + e^{2w}};\ \sin\varphi = e^{w}/b;\ \cos\varphi = \Omega/b.$$

The initial ranges of these amplitude—frequency characteristics are shown in Fig. 1. The amplitude—frequency characteristic of the process response to perturbations of initial

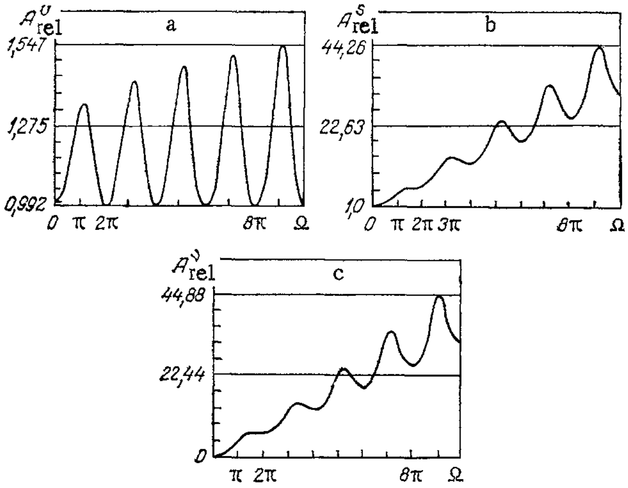

Fig. 1. Amplitude—frequency characteristics of system response, in terms of extruded fiber cross section, to periodic variation of extrusion rate (a), to periodic nonuniformity of initial cross section (b), and to periodic variations of viscosity (c): scale of ordinates 0.0555 (a); 4.326 (b); 4.448 (c).

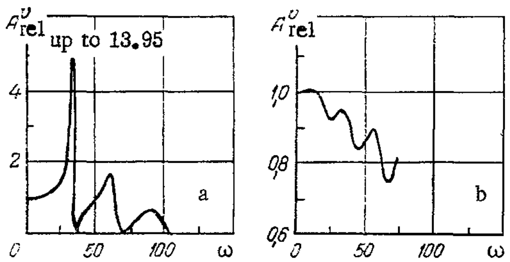

Fig. 2

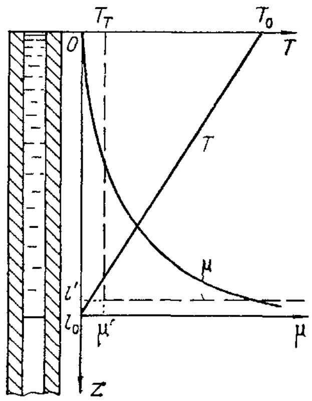

Fig. 3

Fig. 2. Amplitude—frequency characteristics of system response in terms of extruded fiber cross section to perturbations of the extrusion rate in case of nonuniformly distributed viscosity: a) a = 1; b) a = 0.1.

Fig. 3. Schematic diagram depicting extrusion of double-layer fiber.

velocity is not shown here, because it is identical to the corresponding characteristic for perturbation of the extrusion rate.

For the case of a viscosity varying along the z axis we solved Eq. (4) numerically, with the fluidity distribution simulated by the relation

$$\eta(z)=\frac{1}{2\,a\,\mathrm{arctg}\,\dfrac{1}{2a}-\dfrac{a^2}{\dfrac{1}{4}+a^2}}\left[\frac{1}{1+\dfrac{z-\dfrac{1}{2}}{a^2}}-\frac{a^2}{\dfrac{1}{4}+a^2}\right],$$

and the parameter a here characterizing the "slope" of decay of the dimensionless fluidity to zero at the boundary of the deformation zone. The amplitude—frequency characteristics of the system response to perturbation of the extrusion rate with parameter a equal to 1.0 and 0.1, respectively, are shown in Fig. 2.

It must be emphasized, first of all, that the system response to perturbations is of an aftereffect nature with a finite "period of memory lapse" $T = \frac{1}{w}(1 - e^{-w})$. In dimensional form and considering that e^{-w} is smaller than unity, this period will be $T \simeq \mathscr{L}/v_0 \ln v_b/v_0 \simeq 20$ sec for typical values of the process parameters $\mathscr{L} = 15$ cm, $v_o = 0.1$ cm/sec, and $v_b = 100$ cm/sec. This period represents the time in which a material point travels from the beginning to the end of the deformation zone. Naturally, any perturbations of process parameters in a material point located at the beginning of the deformation zone will affect the parameters of extruded fiber as long as this "perturbed" material point remains in the deformation zone and, therefore, a perturbation affects the trend of the process or, more precisely, its trend during the period of T. When the perturbations acting on the system are regular, then regularization of the system response occurs within time T from the instant the "perturbation" has been switched on.

The model of the extrusion process which we propose here describes mathematically this rather obvious physical nature of the system response to perturbations, earlier models [1, 2] having failed to do so.

An analysis of the expressions for the amplitude—frequency characteristics and of the graph in Fig. 1 depicting the initial segments of these characteristics indicates that the system response to perturbations is one with a distinct resonance feature, peaking at frequencies approximately equal to multiples of the inverse time taken by a material point to move through the deformation zone.

These results also lead to conclusions which are valid regardless of the stipulated fluidity distribution. Namely, the system response to perturbations of initial cross section and of fluidity is much stronger than the response to perturbations of the extrusion rate. Furthermore, at low perturbation frequencies the extruded fiber cross section duplicates the perturbations of the extrusion rate and of the original fiber cross section while remaining independent of viscosity perturbations. At high perturbation frequencies, on the other hand, the system response to perturbations of the extrusion rate decays, while the the response to perturbations of the original fiber cross section and of viscosity approaches a finite limit.

Numerical calculations for the case of variable viscosity have revealed that the existence of peaks in the amplitude—frequency characteristics is entirely due to unevenness of the fluidity distribution at the boundaries of the deformation zone and that these peaks vanish as this distribution becomes smoother with a correspondingly decreasing parameter a. They have also revealed that the high-frequency response tends to weaken noticeably as the fluidity distribution becomes smoother.

2. When multilayer optical fibers are extruded, differences between the thermoelastic properties of individual layers produce large residual stresses in the fiber during the cooling process and these stresses strongly influence the strength characteristics of the final fiber. The effect of these stresses is so appreciable that sometimes the strength of a fiber after extrusion eventually drops to nearly zero. Precisely for this reason it is important to know how to calculate the distribution of residual stresses in a fiber and on that basis to determine the strengthwise optimum extrusion mode. Such calculations are usually made according to the standard theory of thermoelasticity, the one-dimensional model being quite adequate in this case of a very large length-to-width ratio. In the extrusion process there often arises a situation, however, where inside the already solidified quartz layer there still remains a not yet solidified mass (quartz glass). In stress calculations for this case one usually assumes that there are no shearing stresses in the liquid and that the latter responds to changes of the cavity volume by changes of pressure only [5]. Considering that the magnitude of this pressure is of the order of 10^3 kgf/cm^2, owing to different thermal expansivities of the liquid core and the solid shell, one can expect that, despite the high viscosity of the liquid and the small inside diameter of the shell, such a large pressure difference can cause viscous flow of the melt inside the shell and this, in turn, will result in a redistribution of pressure.

In this study an attempt was made to estimate the effect of such a viscous stress relaxation during extrusion on the distribution of residual stresses at the instant the fiber has completely solidified. The model of this phenomenon was simplified for this purpose so that it would yield the necessary results in analytical form and, at the same time, rather accurately describe the real process.

We considered extrusion of a double-layer optical fiber during the period from solidification of the shell to solidification of the core. In a real process this corresponds to a change of the temperature from 1800 to 1500°C over the length of a few centimeters (in the case of a quartz shell and a quartz glass core).

The following model of the extrusion process was used for calculations. A tube with perfectly rigid walls and an inside radius r_0 moves along the z axis, which is its own axis, at velocity v_b (Fig. 3). The tube is filled with liquid whose viscosity μ and temperature T are functions of the z coordinate and do not vary in time (in a stationary system of coordinates). The density ρ of the liquid inside this tube depends on pressure p and on temperature T in accordance with Hooke's law and the law of thermal expansion. At distance l_0 from the origin of coordinates the liquid inside the tube solidifies at temperature T_T. There is no tube in the $z < 0$ half-space, which means the tube wall forms all the time at the origin of coordinates. The pressure of liquid at the tube entrance is p_0.

On the basis of this model was formulated the problem

$$\frac{d\tilde{p}}{dz} - \frac{a(z)}{E} = -a(z)\varkappa(T(z) - T_T), \tag{9}$$

where $\tilde{p}(z) = p(z) - p(l_0)$, $a(z) = (24v_b/r_0^2)\mu$; $\varkappa$ is the coefficient of thermal expansion for core material, and E is its modulus of elasticity. The boundary condition is $p(l_0) = 0$. The solution of this problem is

$$p(z) - p(l_0) = \varkappa E\left\{T(z) - T_T + \int_z^{l_0} \frac{dT(x)}{dx} \exp\left[-\int_z^x \frac{a(\lambda)}{E}\, d\lambda\right] dx\right\}. \tag{10}$$

For rough numerical estimates it is permissible to replace the real viscosity distribution (Fig. 3) with a "rectangular" one, as shown with the dashed line on the diagram. Assuming now a linear distribution and small corrections for viscous relaxation, we obtain from relation (10) the estimate

$$\Delta p = p(0) - p(l) = \varkappa E(T(0) - T_T)\left\{1 - \frac{Er_0^2}{24\, v_b \mu' l_0'}\right\}, \tag{11}$$

with parameters μ' and l_0' indicated on the diagram. The effect of viscous relaxation can, accordingly, be estimated as

$$\frac{\Delta p^\infty - \Delta p}{\Delta p^\infty} = \frac{Er_0^2}{24 v_b \mu' l_0'}, \tag{12}$$

where Δp^∞ is the pressure drop with viscous relaxation disregarded and thus with $\mu' = \infty$ in expression (11). For the typical values of parameters in expression (12) $E = 7\cdot10^{10}$ N/m², $r_0 = 10^{-4}$ m, $v_b = 1$ m/sec, $l' = 0.07$ m, $\mu' = 6\cdot10^3$ Pa we estimate the effect of viscous relaxation to be

$$\frac{\Delta p^\infty - \Delta p}{\Delta p^\infty} = 7\,\%.$$

Therefore, under certain conditions of extrusion, the redistribution of residual stresses due to viscous flow of the glass mass in a liquid layer surrounded by solidified ones can become appreciable enough to influence the strength characteristics of the produced fiber.

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